# POLYNOMIALS

## EXERCISE 2.1

**QUESTION 1.** *Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer:*

*(i)* $4x^2 - 3x + 7$ *(ii)* $y^2 + \sqrt{2}$ *(iii)* $3\sqrt{t} + t\sqrt{2}$

*(iv)* $y + \frac{2}{y}$ *(v)* $x^{10} + y^3 + t^{50}$

**SOLUTION.** (*i*) We have, $4x^2 - 3x + 7$

The given expression has single variable $x$.

The index of $x$ is whole number, *i.e.*, 2.

Hence, **the given expression is polynomial in one variable**. **Ans.**

(*ii*) In $y^2 + \sqrt{2}$, the index of $y$ is a whole number, *i.e.*, 2. So it is a **polynomial in one variable $y$.**

(*iii*) We have, $3\sqrt{t} + t\sqrt{2} = 3t^{1/2} + \sqrt{2}\,t$, here the exponent of the first term is $\frac{1}{2}$, which is not a whole number. Therefore, **it is not a polynomial**.

(*iv*) We have, $y + \frac{2}{y} = y + 2y^{-1}$, here the exponent of the second term is $-1$, which is not a whole number and so **it is not a polynomial**.

(*v*) We have, $x^{10} + y^3 + t^{50}$. It is **not a polynomial in one variable** as three variables $x$, $y$, $t$ occur in it.

**QUESTION 2.** *Write the coefficients of $x^2$ in each of the following:*

*(i)* $2 + x^2 + x$ *(ii)* $2 - x^2 + x^3$ *(iii)* $\frac{\pi}{2}x^2 + x$ *(iv)* $\sqrt{2}x - 1$

**SOLUTION.** (*i*) Coefficient of $x^2$: in $2 + x^2 + x$ is **1.**

(*ii*) Coefficient of $x^2$: in $2 - x^2 + x^3$ is **$-1$.**

(*iii*) Coefficient of $x^2$: in $\frac{\pi}{2}x^2 + x$ is $\frac{\pi}{2}$.

(*iv*) Coefficient of $x^2$: in $\sqrt{2}x - 1$ is **0.** **Ans.**

**QUESTION 3.** *Give one example of each of a binomial of degree 35, and of a monomial of degree 100.*

**SOLUTION.** Binomial of degree 35 may be taken as $5x^{35} + 10x$.

Monomial of degree 100 may be taken as $5x^{100}$. **Ans.**

**QUESTION 4.** *Write the degree of each of the following polynomials:*

*(i)* $5x^3 + 4x^2 + 7x$ *(ii)* $4 - y^2$ *(iii)* $5t - \sqrt{7}$ *(iv)* $3$

**SOLUTION.** (*i*) We have, $5x^3 + 4x^2 + 7x$, the highest power term is $5x^3$ and the exponent is 3. So, the **degree is 3**.

(*ii*) We have, $4 - y^2$. The highest power term is $-y^2$ and the exponent is 2. So, the **degree is 2**.

(*iii*) We have, $5t - \sqrt{7}$, the highest power term is $5t$ and the exponent is 1. So, the **degree is 1**.

(*iv*) We have, 3. The only term here is 3 which can be written as $3x$ and so the exponent is 0. Therefore, the **degree is 0**. **Ans.**

**QUESTION 5.** *Classify the following as linear, quadratic and cubic polynomials:*

*(i)* $x^2 + x$ *(ii)* $x - x^3$ *(iii)* $y + y^2 + 4$ *(iv)* $1 + x$ *(v)* $3t$

*(vi)* $r^2$ *(vii)* $7x^3$

**SOLUTION.** (*i*) The highest degree of $x$ in $x^2 + x$ is 2. Hence, it is **quadratic.**

(*ii*) The highest degree of $x$ in $x - x^3$ is 3. Hence, it is **a cubic.**

(*iii*) The highest degree of $y$ in $y + y^2 + 4$ is 2. Hence, it is **quadratic.**

(*iv*) The highest degree of $x$ in $1 + x$ is 1. Hence, it is **a linear polynomial.**

(*v*) The highest degree of $t$ in $3t$ is 1. Hence, it is **a linear polynomial.**

(*vi*) The highest degree of $r$ in $r^2$ is 2. Hence, it is **a quadratic polynomial.**

(*vii*) The highest degree of $x$ in $7x^3$ is 3. Hence, it is **cubic polynomial.** **Ans.**

## EXERCISE 2.2

**QUESTION 1.** *Find the value of the polynomial $5x - 4x^2 + 3$ at*

*(i) $x = 0$* *(ii) $x = -1$* *(iii) $x = 2$*

**SOLUTION.** Let $p(x) = 5x - 4x^2 + 3$ be given polynomial.

(*i*) At $x = 0$; the value of $p(x)$ is,

$$p(0) = 5(0) - 4(0)^2 + 3 = 0 - 0 + 3 = \mathbf{3}$$

(*ii*) At $x = -1$; the value of $p(x)$ is,

$$p(-1) = 5(-1) - 4(-1)^2 + 3 = -5 - 4 + 3 = \mathbf{-6}$$

(*iii*) At $x = 2$; the value of $p(x)$ is,

$$p(2) = 5(2) - 4(2)^2 + 3 = 10 - 16 + 3 = 13 - 16 = \mathbf{-3}.$$ **Ans.**

**QUESTION 2.** *Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials:*

*(i) $p(y) = y^2 - y + 1$* *(ii) $p(t) = 2 + t + 2t^2 - t^3$*

*(iii) $p(x) = x^3$* *(iv) $p(x) = (x - 1)(x + 1)$*

**SOLUTION.** (*i*) We have, $p(y) = y^2 - y + 1$ ...(1)

Putting $y = 0$ in (1), we get

$$p(0) = (0)^2 - 0 + 1 = 0 - 0 + 1 = \mathbf{1},$$

Putting $y = 1$ in (1), we get

$$p(1) = (1)^2 - 1 + 1 = 1 - 1 + 1 = \mathbf{1},$$

Putting $y = 2$ in (1), we get

$$p(2) = (2)^2 - 2 + 1 = 4 - 2 + 1 = \mathbf{3}$$

(*ii*) We have $p(t) = 2 + t + 2t^2 - t^3$ ...(2)

Putting $t = 0$ in (2), we get

$$p(0) = 2 + 0 + 2(0)^2 - (0)^3 = 2 + 0 + 0 + 0 - 0 = \mathbf{2}$$

Putting $t = 1$ in (2), we get

$$p(1) = 2 + 1 + 2(1)^2 - (1)^3 = 2 + 1 + 2 - 1 = 5 - 1 = \mathbf{4}$$

Putting $t = 2$ in (2), we get

$$p(2) = 2 + 2 + 2(2)^2 - (2)^3 = 2 + 2 + 8 - 8 = \mathbf{4}$$ **Ans.**

(*iii*) We have, $p(x) = x^3$ ...(3)

Putting $x = 0$ in (3), we get

$$p(0) = (0)^3 = \mathbf{0}$$

Putting $x = 1$ in (3), we gt

$$p(1) = (1)^3 = \mathbf{1}$$

Putting $x = 2$ in (3), we get

$$p(2) = (2)^3 = \mathbf{8}$$

(*iv*) We have $p(x) = (x - 1)(x + 1)$ ...(4)

Putting $x = 0$ in (4), we get

$$p(0) = (0 - 1)(0 + 1) = (-1)(1) = \mathbf{-1}$$

Putting $x = 1$ in (4), we get

$$p(1) = (1 - 1)(1 + 1) = (0)(2) = \mathbf{0}$$

Putting $x = 2$ in (4), we get

$$p(2) = (2 - 1)(2 + 1) = (1)(3) = \mathbf{3}$$ **Ans.**

**QUESTION 3.** ***Verify whether the following are zeroes of the polynomial, indicated against them,***

*(i)* $p(x) = 3x + 1;\ x = -\frac{1}{3}$

*(ii)* $p(x) = 5x - 4;\ x = \frac{4}{5}$

*(iii)* $p(x) = x^2 - 1;\ x = 1, -1$

*(iv)* $p(x) = (x + 1)(x - 2);\ x = -1, 2$

*(v)* $p(x) = x^2;\ x = 0$

*(vi)* $p(x) = lx + m;\ x = -\frac{m}{l}$

*(vii)* $p(x) = 3x^2 - 1;\ x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

*(viii)* $p(x) = 2x + 1;\ x = \frac{1}{2}$

**SOLUTION.** *(i)* We have, $p(x) = 3x + 1$ ...(1)

Putting $x = -\frac{1}{3}$ in (1), we get

$$p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = \mathbf{0}$$

Hence, $-\frac{1}{3}$ **is a zero of** $p(x)$ **Ans.**

*(ii)* We have, $p(x) = 5x - 4$ ...(2)

Putting $x = \frac{4}{5}$ in (2), we get

$$p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - 4 = 4 - 4 = \mathbf{0}$$

Hence, $\frac{4}{5}$ **is a zero of** $p(x)$. **Ans.**

*(iii)* We have, $p(x) = x^2 - 1$ ...(3)

Putting $x = 1$, in (3), we get

$$p(1) = (1)^2 - 1 = 1 - 1 = \mathbf{0}$$

Hence, **1 is a zero of** $p(x)$.

Also putting $x = -1$ in (3), we get

$$p(-1) = (-1)^2 - 1 = 1 - 1 = \mathbf{0}$$

Hence, **–1 is a zero of** $p(x)$. **Ans.**

*(iv)* We have, $p(x) = (x + 1)(x - 2)$ ...(4)

Putting $x = -1$, in (4), we get

$$p(-1) = (-1 + 1)(-1 - 2) = (0)(-3) = \mathbf{0}$$

Hence, –1 is a zero of $p(x)$.

Also putting $x = 2$, in (4), we get

$$p(2) = (2 + 1)(2 - 2) = (3)(0) = \mathbf{0}$$

Hence, **2 is zero of** $p(x)$. **Ans.**

*(v)* We have, $p(x) = x^2$ ...(5)

Putting $x = 0$, in (5), we get

$$p(0) = (0)^2 = \mathbf{0}$$

Hence, **0 is a zero of** $p(x)$. **Ans.**

*(vi)* We have, $p(x) = lx + m$ ...(6)

Putting $x = -\frac{m}{l}$ in (6), we get

$$p\left(-\frac{m}{l}\right) = l\left(\frac{-m}{l}\right) + m = -m + m = \mathbf{0}$$

Hence, $\frac{-m}{l}$ **is a zero of** $p(x)$. **Ans.**

**(*vii*)** We have, $p(x) = 3x^2 - 1$ ...(7)

Putting $x = -\frac{1}{\sqrt{3}}$ in (7), we get

$$p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 3 \times \frac{1}{3} - 1 = 1 - 1 = \mathbf{0}$$

Hence, $-\frac{1}{\sqrt{3}}$ **is a zero of** $p(x)$.

Also putting $x = \frac{2}{\sqrt{3}}$ in (7), we get

$$p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3 \times \frac{4}{3} - 1 = 4 - 1 = \mathbf{3} \neq \mathbf{0}$$ **Ans.**

Hence, $\frac{2}{\sqrt{3}}$ **is not a zero of** $p(x)$.

**(*viii*)** We have, $p(x) = 2x + 1$ ...(8)

Putting $x = \frac{1}{2}$ in (8), we get

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = \mathbf{2} \neq \mathbf{0}$$

Hence, $\frac{1}{2}$ **is not a zero of** $p(x)$. **Ans.**

**QUESTION 4.** ***Find the zero of the polynomial in each of the following cases :***

***(i)*** $p(x) = x + 5$ ***(ii)*** $p(x) = x - 5$ ***(iii)*** $p(x) = 2x + 5$

***(iv)*** $p(x) = 3x - 2$ ***(v)*** $p(x) = 3x$ ***(vi)*** $p(x) = ax, a \neq 0$

***(vii)*** $p(x) = cx + d, c \neq 0$, *c, d are real numbers.*

**SOLUTION.** **(*i*)** We have to solve $p(x) = 0$

or $x + 5 = 0 \Rightarrow x = -\mathbf{5}$

∴ – 5 is a zero of the polynomial $x + 5$.

**(*ii*)** We have to solve $p(x) = 0$

or $x - 5 = 0 \Rightarrow x = \mathbf{5}$

∴ 5 is a zero of the polynomial $x - 5$.

**(*iii*)** We have to solve $p(x) = 0$

or $2x + 5 = 0 \Rightarrow \boldsymbol{x = -\frac{5}{2}}$

∴ $-\frac{5}{2}$ is a zero of the polynomial $2x + 5$.

**(*iv*)** We have to solve $p(x) = 0$

or $3x - 2 = 0 \Rightarrow \boldsymbol{x = \frac{2}{3}}$

∴ $\frac{2}{3}$ is a zero of the polynomial $3x - 2$.

**(*v*)** We have to solve $p(x) = 0$

or $3x = 0 \Rightarrow x = \mathbf{0}$

∴ 0 is a zero of the polynomial $3x$.

**(*vi*)** We have to solve $p(x) = ax, a \neq 0$

or $ax = 0$ or $x = \mathbf{0}$

∴ 0 is a zero of the polynomial $ax$.

**(*vii*)** We have to solve $p(x) = 0, c \neq 0$

or $cx + d = 0 \Rightarrow \boldsymbol{x = -\frac{d}{c}}$

∴ $-\frac{d}{c}$ is a zero of the polynomial $cx + d$. **Ans.**

## EXERCISE 2.3

**QUESTION 1.** ***Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by***

***(i)*** $x + 1$ ***(ii)*** $x - \frac{1}{2}$ ***(iii)*** $x$ ***(iv)*** $x + \pi$ ***(v)*** $5 + 2x$

**SOLUTION.** **(*i*)** By remainder theorem, the required remainder is equal to $p(-1)$.

Now, $p(x) = x^3 + 3x^2 + 3x + 1$

$\therefore$ $p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1 = -1 + 3 - 3 + 1 = 0$

Hence, required remainder = $p(-1)$ = **0**.

**(*ii*)** By remainder theorem, the required remainder is equal to $p\left(\frac{1}{2}\right)$.

Now, $p(x) = x^3 + 3x^2 + 3x + 1$

$$\therefore \quad p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$$

$$= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1 = \frac{1 + 6 + 12 + 8}{8} = \mathbf{\frac{27}{8}}$$

**(*iii*)** By remainder theorem, the required remainder is equal to $p(0)$.

Now, $p(x) = x^3 + 3x^2 + 3x + 1$

$\therefore$ $p(0) = 0 + 0 + 0 + 1 = 1$

Hence, the required remainder = $p(0)$ = **1**.

**(*iv*)** By remainder theorem, the required remainder is $p(-\pi)$.

Now, $p(x) = x^3 + 3x^2 + 3x + 1$

$\therefore$ Remainder = $p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$

$= \mathbf{-\pi^3 + 3\pi^2 - 3\pi + 1}$

**(*v*)** By remainder theorem, the required remainder is $p\left(-\frac{5}{2}\right)$.

Now, $p(x) = x^3 + 3x^2 + 3x + 1$

$$\therefore \quad p\left(-\frac{5}{2}\right) = \left(-\frac{5}{2}\right)^3 + 3\left(\frac{-5}{2}\right)^2 + 3\left(\frac{-5}{2}\right) + 1$$

$$= -\frac{125}{8} + \frac{75}{4} - \frac{15}{2} + 1 = \frac{-125 + 150 - 60 + 8}{8} = \mathbf{\frac{-27}{8}}$$ **Ans.**

**QUESTION 2.** ***Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.***

**SOLUTION.** Let $p(x) = x^3 - ax^2 + 6x - a$ ...(1)

By remainder theorem, when $p(x)$ is divided by $x - a$. Then remainder = $p(a)$.

Putting $x = a$ in (1), we get

$\therefore$ $p(a) = a^3 - a \,.\, a^2 + 6a - a$

$= a^3 - a^3 + 6a - a = \mathbf{5a}$

Hence, the required remainder is $p(a) = 5a$. **Ans.**

**QUESTION 3.** ***Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.***

**SOLUTION.** Let $p(x) = 3x^3 + 7x$ ...(1)

Now, $7 + 3x = 0 \Rightarrow x = -\frac{7}{3}$

$7 + 3x$ will be a factor of $p(x) = 3x^3 + 7x$ if $p\left(-\frac{7}{3}\right) = 0$

Putting $x = -\frac{7}{3}$ in (1), we get

$$p\left(-\frac{7}{3}\right) = 3\left(-\frac{7}{3}\right)^3 + 7\left(-\frac{7}{3}\right)$$

$$= 3\left(\frac{-343}{27}\right) - \frac{49}{3} = -\frac{343}{9} - \frac{49}{3} = -\frac{490}{9} \neq 0$$

Hence, $7 + 3x$ is not a factor of $\mathbf{3x^3 + 7x}$. **Ans.**

## EXERCISE 2.4

**QUESTION 1.** *Determine which of the following polynomials has (x + 1) as a factor :*

*(i)* $x^3 + x^2 + x + 1$ *(ii)* $x^4 + x^3 + x^2 + x + 1$

*(iii)* $x^4 + 3x^3 + 3x^2 + x + 1$ *(iv)* $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

**SOLUTION.** ***(i)*** Let $f(x) = x^3 + x^2 + x + 1$ ...(1)

By factor theorem, $(x + 1)$ will be a factor of $f(x)$, if $f(-1) = 0$

On putting $x = -1$ in (1), we get

$$f(-1) = (-1)^3 + (-1)^2 + (-1) + 1 = -1 + 1 - 1 + 1 = 0$$

Hence, $x + 1$ is a factor of $\mathbf{x^3 + x^2 + x + 1}$. **Ans.**

***(ii)*** Let $p(x) = x^4 + x^3 + x^2 + x + 1$ ...(1)

In order to prove that $(x + 1)$ is a factor of (1).

$x + 1 = 0 \Rightarrow x = -1$. Then it is sufficient to show that $p(-1) = 0$.

Putting $x = -1$ in (1), we get

$$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$$
$$= 1 - 1 + 1 - 1 + 1 = 1 \neq 0$$

Hence, $(x + 1)$ is not a factor of $\mathbf{x^4 + x^3 + x^2 + x + 1}$. **Ans.**

***(iii)*** Let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$ ...(2)

In order to prove that $(x + 1)$ is a factor of (2)

$x + 1 = 0 \Rightarrow x = -1$. Then it is sufficient to show that $p(-1) = 0$.

Putting $x = -1$ in (2), we get

$$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$$
$$= 1 - 3 + 3 - 1 + 1 = 1 \neq 0$$

Hence, $(x + 1)$ is not a factor of $\mathbf{x^4 + 3x^3 + 3x^2 + x + 1}$. **Ans.**

***(iv)*** Let $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$ ...(3)

In order to prove that $(x + 1)$ is a factor of (3).

$x + 1 = 0 \Rightarrow x = -1$. Then it is sufficient to show that $p(-1) = 0$.

Putting $x = -1$ in (3), we get

$$p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$$
$$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2} = 2\sqrt{2} \neq 0$$

Hence, $(x + 1)$ is not a factor of $\mathbf{x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}}$. **Ans.**

**QUESTION 2.** *Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:*

*(i)* $p(x) = 2x^3 + x^2 - 2x - 1,\ g(x) = x + 1$

*(ii)* $p(x) = x^3 + 3x^2 + 3x + 1,\ g(x) = x + 2$

*(iii)* $p(x) = x^3 - 4x^2 + x - 6,\ g(x) = x - 3$

**SOLUTION.** ***(i)*** We have, $p(x) = 2x^3 + x^2 - 2x - 1$ ...(1)

In order to prove that $g(x) = x + 1$ is a factor of (1).

$$x + 1 = 0 \Rightarrow x = -1$$

then it is sufficient to show that $p(-1) = 0$

Putting $x = -1$ in (1), we get

$$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$$
$$= -2 + 1 + 2 - 1 = 0$$

Hence, **$g(x)$ is a factor of $p(x)$.** **Ans.**

(*ii*) We have, $p(x) = x^3 + 3x^2 + 3x + 1$ ...(2)

In order to prove that $g(x) = x + 2$ is a factor of (2).

$x + 2 = 0 \Rightarrow x = -2$ then it is sufficient to show that $p(-2) = 0$

Putting $x = -2$ in (2), we get

$$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$$
$$= -8 + 12 - 6 + 1 = -1 \neq 0$$

Hence, **$g(x)$ is not a factor of $p(x)$.** **Ans.**

(*iii*) We have, $p(x) = x^3 - 4x^2 + x - 6$ ...(3)

In order to prove that $g(x) = x - 3$ is a factor of (3).

$x - 3 = 0 \Rightarrow x = 3$ then it is sufficient to show that $p(3) = 0$.

Putting $x = 3$ in (3), we get

$$p(3) = (3)^3 - 4(3)^2 + 3 - 6$$
$$= 27 - 36 + 3 - 6 = -12 \neq 0$$

Hence, **$g(x)$ is not a factor of $p(x)$.** **Ans.**

**QUESTION 3.** ***Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:***

*(i)* $p(x) = x^2 + x + k$ *(ii)* $p(x) = 2x^2 + kx + \sqrt{2}$

*(iii)* $p(x) = kx^2 - \sqrt{2}x + 1$ *(iv)* $p(x) = kx^2 - 3x + k$.

**SOLUTION.** (*i*) We have, $p(x) = x^2 + x + k$

If $(x - 1)$ is a factor of $p(x) = x^2 + x + k$, then

$$p(1) = 0$$
$$\Rightarrow (1)^2 + 1 + k = 0$$
$$\Rightarrow 1 + 1 + k = 0$$
$$\Rightarrow k = -2$$

Hence, $\boldsymbol{k = -2}$ **Ans.**

(*ii*) We have, $p(x) = 2x^2 + kx + \sqrt{2}$

If $(x - 1)$ is a factor of $p(x) = 2x^2 + kx + \sqrt{2}$, then

$$p(1) = 0$$
$$\Rightarrow 2(1)^2 + k(1) + \sqrt{2} = 0 \Rightarrow 2 + k + \sqrt{2} = 0$$
$$\Rightarrow k = -(2 + \sqrt{2})$$

Hence, $\boldsymbol{k = -(2 + \sqrt{2})}$ **Ans.**

(*iii*) We have, $p(x) = kx^2 - \sqrt{2}x + 1$

If $(x - 1)$ is a factor of $p(x) = kx^2 - \sqrt{2}x + 1$, then

$$p(1) = 0$$
$$\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0 \Rightarrow k - \sqrt{2} + 1 = 0$$

Hence, $\boldsymbol{k = \sqrt{2} - 1}$ **Ans.**

(*iv*) We have, $p(x) = kx^2 - 3x + k$

If $(x - 1)$ is a factor of $p(x) = kx^2 - 3x + k$, then

$$p(1) = 0$$
$$\Rightarrow k(1)^2 - 3(1) + k = 0 \Rightarrow k - 3 + k = 0$$

$\Rightarrow \qquad 2k = 3 \quad \Rightarrow \quad k = \frac{3}{2}$

Hence, $\boldsymbol{k = \frac{3}{2}}$ **Ans.**

**QUESTION 4.** *Factorize :*

*(i)* $12x^2 - 7x + 1$ *(ii)* $2x^2 + 7x + 3$ *(iii)* $6x^2 + 5x - 6$ *(iv)* $3x^2 - x - 4$

**SOLUTION.** **(*i*)** $12x^2 - 7x + 1$

Here $a = 12, b = -7, c = 1$ [*Standard form* $= ax^2 + bx + c$]

**(*a*)** To factorise the given quadratic we have to find $p$ and $q$ such that

$$p + q = b \qquad pq = ac$$

*i.e.,* $p + q = -7 \qquad pq = 12 \times 1 = 12$

**(*b*)** We have to find out two factors of 12 such that their sum = –7

By trial $-4 - 3 = -7, (-4) \times (-3) = 12$

**(*c*)** Split the middle term $-7x = -4x - 3x$

**(*d*)** Factorise by grouping

$$12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$$
$$= 4x(3x - 1) - 1(3x - 1) = \mathbf{(3x - 1)(4x - 1)}$$ **Ans.**

**(*ii*)** We have, $2x^2 + 7x + 3$

Here $a = 2, b = 7, c = 3$ [*Standard form* $= ax^2 + bx + c$]

**(*a*)** To factorize the given quadratic we have to find $p$ and $q$ such that

$$p + q = b \qquad pq = ac$$

*i.e.,* $p + q = 7 \qquad$ *i.e.,* $pq = 2 \times 3 = 6$

**(*b*)** We have to find out two factors of 6 such that their sum is 7.

By trial $1 + 6 = 7, 1 \times 6 = 6$

**(*c*)** Split the middle term

$$7x = x + 6x, \quad 6 = 1 \times 6$$

**(*d*)** Factorise by grouping

$$2x^2 + 7x + 3 = 2x^2 + x + 6x + 3$$
$$= x(2x + 1) + 3(2x + 1) = \mathbf{(2x + 1)(x + 3)}$$ **Ans.**

**(*iii*)** We have, $6x^2 + 5x - 6$

Here, $a = 6, b = 5, c = -6$ [*Standard form* $= ax^2 + bx + c$]

**(*a*)** To factorise the given quadratic we have to find $p$ and $q$ such that

$$p + q = b \qquad pq = ac \quad \Rightarrow \quad pq = 6(-6) = -36$$

$\Rightarrow \qquad p + q = 5$

**(*b*)** We have to find out two factors of –36 such that their difference is 5.

By trial, $9 + (-4) = 9 - 4 = 5, 9 \times (-4) = -36$

**(*c*)** Split the middle term

$$5x = 9x - 4x, -36 = 9 \times (-4)$$

**(*d*)** Factorise by grouping

$$6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6 = 3x(2x + 3) - 2(2x + 3)$$
$$= \mathbf{(2x + 3)(3x - 2)}$$ **Ans.**

**(*iv*)** We have, $3x^2 - x - 4$

Here, $a = 3, b = -1, c = -4$ [*Standard form* $= ax^2 + bx + c$]

**(*a*)** To factorise the given quadratic we have to find $p$ and $q$ such that

$$p + q = b \qquad pq = ac$$

*i.e.,* $p + q = -1 \qquad \Rightarrow pq = (3)(-4) = -12$

**(*b*)** We have to find out two factors of –12 such that their sum is –1.

By trial $3 + (-4) = -1, 3 \times (-4) = -12$

**(*c*)** Split the middle term

$$-x = 3x - 4x, \quad -12 = 3 \times (-4)$$

(*d*) Factorise by grouping

$$3x^2 - x - 4 = 3x^2 + 3x - 4x - 4$$
$$= 3x(x + 1) - 4(x + 1) = \mathbf{(x + 1)(3x - 4)}$$ **Ans.**

**QUESTION 5.** ***Factorise:***

*(i)* $x^3 - 2x^2 - x + 2$ *(ii)* $x^3 - 3x^2 - 9x - 5$

*(iii)* $x^3 + 13x^2 + 32x + 20$ *(iv)* $2y^3 + y^2 - 2y - 1$

**SOLUTION.** (*i*) Let $p(x) = x^3 - 2x^2 - x + 2$

The constant term of $p(x)$ is 2.

The factors of the constant term 2 are ±1, ±2

Putting $x = 1$ in $p(x)$, we have

$$p(1) = (1)^3 - 2(1)^2 - 1 + 2 = 1 - 2 - 1 + 2 = 0$$

∴ $(x - 1)$ is a factor of $p(x)$.

Putting $x = -1$ in $p(x)$, we have

$$p(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2 = -1 - 2 + 1 + 2 = 0$$

∴ $x + 1$ is a factor of $p(x)$.

Putting $x = 2$ in $p(x)$, we have

$$p(2) = (2)^3 - 2(2)^2 - (2) + 2 = 8 - 8 - 2 + 2 = 0$$

∴ $(x + 2)$ is a factor of $p(x)$.

Since cubic so can have only three factor.

∴ The factors of $p(x)$ are $(x - 1)$, $(x + 1)$ and $(x - 2)$.

Let $p(x) = k(x - 1)(x + 1)(x - 2)$

⇒ $x^3 - 2x^2 - x + 2 = k(x - 1)(x + 1)(x - 2)$ ...(1)

Putting $x = 0$ on both sides, we have

$$2 = k(-1)(1)(-2) \Rightarrow k = 1$$

Putting $k = 1$ in (1), we get

∴ $x^3 - 2x^2 - x + 2 = \mathbf{(x - 1)(x + 1)(x - 2)}$ **Ans.**

(*ii*) Let $p(x) = x^3 - 3x^2 - 9x - 5$ ...(1)

The constant term in $p(x)$ is –5 and its factors are ±1, ±5.

On putting $x = -1$ in (1), we get

$$p(-1) = -1 - 3 + 9 - 5 = 0$$

So, $(x + 1)$ is a factor of $p(x)$.

Now, we divide $p(x) = x^3 - 3x^2 - 9x - 5$ by $(x + 1)$ to get other factors.

$$\begin{array}{r|l} & x^2 - 4x - 5 \\ \hline x + 1 & x^3 - 3x^2 - 9x - 5 \\ & x^3 + x^2 \\ & - \quad - \\ \hline & -4x^2 - 9x \\ & -4x^2 - 4x \\ & + \quad + \\ \hline & -5x - 5 \\ & -5x - 5 \\ & + \quad + \\ \hline & 0 \end{array}$$

Thus, $x^3 - 3x^2 - 9x - 5 = (x + 1)(x^2 - 4x - 5) = (x + 1)(x^2 + x - 5x - 5)$

$= (x + 1)[x(x + 1) - 5(x + 1)] = (x + 1)(x + 1)(x - 5)$

Hence, $x^3 - 3x^2 - 9x - 5 = \mathbf{(x + 1)(x + 1)(x - 5)}$ **Ans.**

(*iii*) Let $p(x) = x^3 + 13x^2 + 32x + 20$ ...(1)

The constant term in $p(x)$ is 20 and its factors are ±1, ±2, ±4, ±5, ±10 and ±20.

On putting $x = -2$ in (1), we get

$p(-2) = -8 + 52 - 64 + 20 = 0$

$(x + 2)$ is a factor of $p(x)$.

Now, divide $p(x)$ by $x + 2$ to get other factors :

$$
\begin{array}{r|l}
 & x^2 + 11x + 10 \\
x + 2 & x^3 + 13x^2 + 32x + 20 \\
 & x^3 + 2x^2 \\
 & - \quad - \\
\hline
 & 11x^2 + 32x \\
 & 11x^2 + 22x \\
 & - \quad - \\
\hline
 & 10x + 20 \\
 & 10x + 20 \\
 & - \quad - \\
\hline
 & 0
\end{array}
$$

Thus, $x^3 + 13x^2 + 32x + 20 = (x + 2)(x^2 + 11x + 10) = (x + 2)(x^2 + x + 10x + 10)$

$= (x + 2)[x(x + 1) + 10(x + 1)] = \mathbf{(x + 2)(x + 1)(x + 10)}$ **Ans.**

(*iv*) Let $p(y) = 2y^3 + y^2 - 2y - 1$ ...(1)

The constant term in $p(y)$ is 1 and its factors are ±1.

On putting $y = 1$ in (1), we get

$p(1) = 2 + 1 - 2 - 1 = 0.$

So, $(y - 1)$ is a factor of $p(y)$.

Now, we divide $p(y)$ by $(y - 1)$ to get other factors.

$$
\begin{array}{r|l}
 & 2y^2 + 3y + 1 \\
y - 1 & 2y^3 + y^2 - 2y - 1 \\
 & 2y^3 - 2y^2 \\
 & - \quad + \\
\hline
 & 3y^2 - 2y \\
 & 3y^2 - 3y \\
 & - \quad + \\
\hline
 & y - 1 \\
 & y - 1 \\
\hline
 & 0
\end{array}
$$

Thus, $2y^3 + y^2 - 2y - 1 = (y - 1)(2y^2 + 3y + 1) = (y - 1)(2y^2 + 2y + y + 1)$

$= (y - 1)[2y(y + 1) + 1(y + 1)] = \mathbf{(y - 1)(y + 1)(2y + 1)}$ **Ans.**

## EXERCISE 2.5

**QUESTION 1.** *Use suitable identities to find the following products :*

*(i)* $(x + 4)(x + 10)$ *(ii)* $(x + 8)(x - 10)$ *(iii)* $(3x + 4)(3x - 5)$

*(iv)* $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$ *(v)* $(3 - 2x)(3 + 2x)$

**SOLUTION.** *(i)* $(x + 4)(x + 10) = x^2 + (4 + 10)x + 4 \times 10$
$= \mathbf{x^2 + 14x + 40}$

*(ii)* $(x + 8)(x - 10) = x^2 + (8 - 10)x + 8 \times (-10)$
$= \mathbf{x^2 - 2x - 80}$

*(iii)* $(3x + 4)(3x - 5) = (3x)^2 + (4 - 5)3x + 4 \times (-5) = \mathbf{9x^2 - 3x - 20}$
[*Using* $(x + a)(x + b) = x^2 + (a + b)x + ab$. *Here, x is taken as* $3x$]

*(iv)* $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right) = (y^2)^2 - \left(\frac{3}{2}\right)^2 = \mathbf{y^4 - \frac{9}{4}}$

*(v)* $(3 - 2x)(3 + 2x) = (3)^2 - (2x)^2 = \mathbf{9 - 4x^2}$ **Ans.**

**QUESTION 2.** *Evaluate the following products without multiplying directly:*

*(i) 103 × 107* *(ii) 95 × 96* *(iii) 104 × 96*

**SOLUTION.** *(i)* $103 \times 107 = (100 + 3) \times (100 + 7)$
$= 100^2 + (3 + 7) \times 100 + 3 \times 7$
$= 10000 + 1000 + 21 = \mathbf{11021}$ **Ans.**

*(ii)* $95 \times 96 = (100 - 5) \times (100 - 4)$
$= (100)^2 - (4 + 5)100 + 5 \times 4$
$= 10000 - 900 + 20 = \mathbf{9120}$ **Ans.**

*(iii)* $104 \times 96 = (100 + 4) \times (100 - 4)$
$= (100)^2 - (4)^2 = 10000 - 16 = \mathbf{9984}$ **Ans.**

**QUESTION 3.** *Factorise the following using appropriate identities :*

*(i)* $9x^2 + 6xy + y^2$ *(ii)* $4y^2 - 4y + 1$ *(iii)* $x^2 - \frac{y^2}{100}$

**SOLUTION.** *(i)* $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)(y) + (y)^2$
$= (3x + y)^2 = \mathbf{(3x + y)(3x + y)}$

*(ii)* $4y^2 - 4y + 1 = (2y)^2 - 2(2y)(1) + (1)^2$
$= (2y - 1)^2 = \mathbf{(2y - 1)(2y - 1)}$

*(iii)* $x^2 - \frac{y^2}{100} = (x)^2 - \left(\frac{y}{10}\right)^2 = \left(\mathbf{x - \frac{y}{10}}\right)\left(\mathbf{x + \frac{y}{10}}\right)$ **Ans.**

**QUESTION 4.** *Expand each of the following, using suitable identities :*

*(i)* $(x + 2y + 4z)^2$ *(ii)* $(2x - y + z)^2$ *(iii)* $(-2x + 3y + 2z)^2$

*(iv)* $(3a - 7b - c)^2$ *(v)* $(-2x + 5y - 3z)^2$ *(vi)* $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

**SOLUTION.** *(i)* We have, $(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$
$[(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$
$= \mathbf{x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx}$

*(ii)* We have, $(2x - y + z)^2 = [2x + (-y) + z]^2$ $[\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$

$= (2x)^2 + (-y)^2 + z^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)$

$= \mathbf{4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx}$

***(iii)*** We have, $(-2x + 3y + 2z)^2 = [(-2x) + 3y + 2z]^2$ $[(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$

$= (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 3(2z)(-2x)$

$= \mathbf{4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx}$

***(iv)*** We have, $(3a - 7b - c)^2 = [3a + (-7b) + (-c)]^2$ $[\because (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$

$= (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a)$

$= \mathbf{9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca}$

***(v)*** We have, $(-2x + 5y - 3z)^2 = [(-2x) + 5y + (-3z)]^2$ $[(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$

$= (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-3z)(-2x)$

$= \mathbf{4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx}$

***(vi)*** We have, $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2 = \left[\frac{1}{4}a + \left(-\frac{1}{2}b\right) + 1\right]^2$ $[(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$

$= \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right) + 2\left(-\frac{1}{2}b\right)(1) + 2(1)\left(\frac{1}{4}a\right)$

$= \mathbf{\frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a}$ **Ans.**

**QUESTION 5.** *Factorise :*

*(i)* $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$ *(ii)* $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\,xy + 4\sqrt{2}\,yz - 8xz$

**SOLUTION.** *(i)* We have, $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(2x)(-4z)$

$[a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2]$

$= [2x + 3y + (-4z)]^2 = \mathbf{(2x + 3y - 4z)^2}$

***(ii)*** We have, $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\,xy + 4\sqrt{2}\,yz - 8xz$

$= (\sqrt{2}\,x)^2 + (-y)^2 + (-2\sqrt{2}\,z)^2 + 2(\sqrt{2}\,x)(-y) + 2(-y)(-2\sqrt{2}\,z) + 2(\sqrt{2}\,x)(-2\sqrt{2}\,z)$

$= [\sqrt{2}\,x + (-y) + (-2\sqrt{2}\,z)]^2 = \mathbf{(\sqrt{2}\,x - y - 2\sqrt{2}z)^2}$ **Ans.**

**QUESTION 6.** *Write the following cubes in expanded form :*

*(i)* $(2x + 1)^3$ *(ii)* $(2a - 3b)^3$ *(iii)* $\left[\frac{3}{2}x + 1\right]^3$ *(iv)* $\left[x - \frac{2}{3}y\right]^3$

**SOLUTION.** *(i)* We have, $(2x + 1)^3 = (2x)^3 + 3(2x)^2(1) + 3(2x)(1)^2 + (1)^3$ $[(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3]$

$= \mathbf{8x^3 + 12x^2 + 6x + 1}$

***(ii)*** We have, $(2a - 3b)^3 = (2a)^3 + 3(2a)^2(-3b) + 3(2a)(-3b)^2 + (-3b)^3$

$= \mathbf{8a^3 - 36a^2b + 54ab^2 - 27b^3}$

***(iii)*** We have, $\left[\frac{3}{2}x + 1\right]^3 = \left(\frac{3}{2}x\right)^3 + 3\left(\frac{3}{2}x\right)^2(1) + 3\left(\frac{3}{2}x\right)(1)^2 + 1^3$

$= \mathbf{\frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1}$

**(*iv*)** We have, $\left[x-\frac{2}{3}y\right]^3 = x^3+3(x)^2\left(-\frac{2}{3}y\right)+3(x)\left(-\frac{2}{3}y\right)^2+\left(-\frac{2}{3}y\right)^3$

$= \mathbf{x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3}$ **Ans.**

**QUESTION 7.** ***Evaluate the following using suitable identities:***

***(i) (99)³*** ***(ii) (102)³*** ***(iii) (998)³***

**SOLUTION.** **(*i*)** $(99)^3 = (100-1)^3$

$= (100)^3 - 3\times 100\times 1(100-1)-(1)^3$ $[(a-b)^3 = a^3 - 3ab(a-b) - b^3]$

$= 1000,000 - 300\times 99 - 1$

$= 1000,000 - 29700 - 1 = 1000,000 - 29701 = \mathbf{970299}$

**(*ii*)** $(102)^3 = (100+2)^3 = (100)^3 + 3\times 100\times 2(100+2)+(2)^3$

$= (1000,000 + 600\ (102) + 8$

$= 1000,000 + 61200 + 8 = 1000,000 + 61208 = \mathbf{1061208}$ **Ans.**

**(*iii*)** $(998)^3 = (1000-2)^3$

$= (1,000)^3 - 3\times 1000\times 2(1000-2)-(2)^3$

$= 1,00,00,00,000 - 6000\times 998 - 8$

$= 1,00,00,00,000 - 5988000 - 8 = 1,00,00,00,000 - 5988008$

$= \mathbf{994011992}$ **Ans.**

**QUESTION 8.** ***Factorise each of the following :***

***(i)*** $8a^3 + b^3 + 12a^2b + 6ab^2$ ***(ii)*** $8a^3 - b^3 - 12a^2b + 6ab^2$

***(iii)*** $27 - 125a^3 - 135a + 225a^2$ ***(iv)*** $64a^3 - 27b^3 - 144a^2b + 108ab^2$

***(v)*** $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

**SOLUTION.** **(*i*)** We have, $8a^3 + b^3 + 12a^2b + 6ab^2 = (2a)^3 + (b)^3 + 3(2a)(b)(2a+b)$

$[a^3 + b^3 + 3ab(a+b) = (a+b)^3]$

$= (2a+b)^3 = \mathbf{(2a+b)(2a+b)(2a+b)}$

**(*ii*)** We have, $8a^3 - b^3 - 12a^2b + 6ab^2 = (2a)^3 + (-b)^3 + 3(2a)(-b)(2a-b)$

$= (2a-b)^3 = \mathbf{(2a-b)(2a-b)(2a-b)}$

**(*iii*)** We have, $27 - 125a^3 - 135a + 225a^2 = (3)^3 + (-5a)^3 + 3(3)(-5a)(3-5a)$

$= (3-5a)^3 = \mathbf{(3-5a)(3-5a)(3-5a)}$

**(*iv*)** We have, $64a^3 - 27b^3 - 144a^2b + 108ab^2 = (4a)^3 + (-3b)^3 + 3(4a)(-3b)(4a-3b)$

$= (4a-3b)^3 = \mathbf{(4a-3b)(4a-3b)(4a-3b)}$

**(*v*)** We have, $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p = (3p)^3 + \left(-\frac{1}{6}\right)^3 + 3(3p)\left(-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)$

$= \left(3p-\frac{1}{6}\right)^3 = \mathbf{\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)}$ **Ans.**

**QUESTION 9.** ***Verify : (i)*** $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

***(ii)*** $x^3 - y^3 = (x-y)\ (x^2 + xy + y^2)$

**SOLUTION.** **(*i*)** R.H.S. $= (x+y)(x^2 - xy + y^2)$

$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$

$= x^3 + y^3 =$ L.H.S. **Proved.**

***(ii)*** R.H.S. $= (x - y)(x^2 + xy + y^2)$

$= x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$

$= x^3 - y^3 =$ L.H.S. **Proved.**

**QUESTION 10.** ***Factorise each of the following :***

***(i)*** $27y^3 + 125z^3$ ***(ii)*** $64m^3 - 343n^3$

**SOLUTION.** (*i*) We have, $27y^3 + 125z^3 = (3y)^3 + (5z)^3$ $[a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

$= (3y + 5z)[(3y)^2 - (3y)(5z) + (5z)^2]$

$= \mathbf{(3y + 5z)(9y^2 - 15yz + 25z^2)}$

(*ii*) We have, $64m^3 - 343n^3 = (4m)^3 + (-7n)^3$ $[a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

$= (4m - 7n)[(4m)^2 - (4m)(-7n) + (-7n)^2]$

$= \mathbf{(4m - 7n)(16m^2 + 28mn + 49n^2)}$ **Ans.**

**QUESTION 11.** ***Factorise :*** $27x^3 + y^3 + z^3 - 9xyz$

**SOLUTION.** $27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3(3x)(y)(z)$

$= (3x + y + z)[(3x)^2 + y^2 + z^2 - (3x)y - yz - z(3x)]$

$= \mathbf{(3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)}$ **Ans.**

**QUESTION 12.** ***Verify that*** $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

**SOLUTION.** L.H.S. $= \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= \frac{1}{2}(x + y + z)(x^2 - 2xy + y^2 + y^2 - 2yz + z^2 + z^2 - 2zx + x^2)$

$= (x + y + z)(x^2 + y^2 + z^2 - yz - zx - xy)$

$= x^3 + y^3 + z^3 - 3xyz$

$=$ R.H.S. **Verified.**

**QUESTION 13.** ***If*** $x + y + z = 0$, ***show that*** $x^3 + y^3 + z^3 = 3xyz$.

**SOLUTION.** We have, $x + y + z = 0$

$\Rightarrow$ $x + y = -z$ ...(1)

Cubing both sides of (1), we have

$(x + y)^3 = (-z)^3$

$\Rightarrow$ $x^3 + y^3 + 3xy(x + y) = -z^3$

$\Rightarrow$ $x^3 + y^3 + 3xy(-z) = -z^3$ $[\because x + y = -z]$

$\Rightarrow$ $x^3 + y^3 + z^3 = \mathbf{3xyz}$ **Proved.**

**QUESTION 14.** ***Without actually calculating the cubes, find the values of each of the following:***

***(i)*** $(-12)^3 + (7)^3 + (5)^3$ ***(ii)*** $(28)^3 + (-15)^3 + (-13)^3$

**SOLUTION.** (*i*) Using the formula $a^3 + b^3 + c^3 = 3abc$, if $a + b + c = 0$.

Here $a = -12$, $b = 7$ and $c = 5$

$\Rightarrow$ $a + b + c = -12 + 7 + 5 = 0$

$\Rightarrow$ $(-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5) = \mathbf{-1260}$

(*ii*) Here, $a = 28$, $b = -15$, and $c = -13$

$\Rightarrow$ $a + b + c = 28 - 15 - 13 = 0$

$\Rightarrow$ $(28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13) = \mathbf{16380}$ **Ans.**

**QUESTION 15.** ***Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given***

***(i) Area : $25a^2 - 35a + 12$ (ii) Area : $35y^2 + 13y - 12$***

**SOLUTION.** Possible length and breadth of the rectangle are the factors of its given area.

**(*i*)** $$\text{Area} = 25a^2 - 35a + 12 = 25a^2 - 15a - 20a + 12$$
$$= 5a(5a - 3) - 4(5a - 3) = (5a - 3)(5a - 4)$$

Hence, possible length and breadth are **$(5a - 3)$ and $(5a - 4)$ units.**

**(*ii*)** $$\text{Area} = 35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$$
$$= 7y(5y + 4) - 3(5y + 4) = (5y + 4)(7y - 3)$$

Hence, possible length and breadth are **$(5y + 4)$ and $(7y - 3)$ units.** **Ans.**

**QUESTION 16.** ***What are the possible expressions for the dimensions of the cuboids whose volumes are given below:***

***(i) Volume : $3x^2 - 12x$ (ii) Volume : $12ky^2 + 8ky - 20k$***

**SOLUTION.** Possible expressions for the dimensions of the cuboids are the factors of their volumes.

**(*i*)** $$\text{Volume} = 3x^2 - 12x = 3x(x - 4)$$

Hence, possible dimensions of cuboid are **3, $x$ and $(x - 4)$ units.**

**(*ii*)** $$\text{Volume} = 12ky^2 + 8ky - 20k = 4k(3y^2 + 2y - 5)$$
$$= 4k(3y^2 - 3y + 5y - 5)$$
$$= 4k[3y(y - 1) + 5(y - 1)] = 4k(y - 1)(3y + 5)$$

Hence, possible dimensions of cuboid are **$4k$, $(y - 1)$ and $(3y + 5)$ units.** **Ans.**