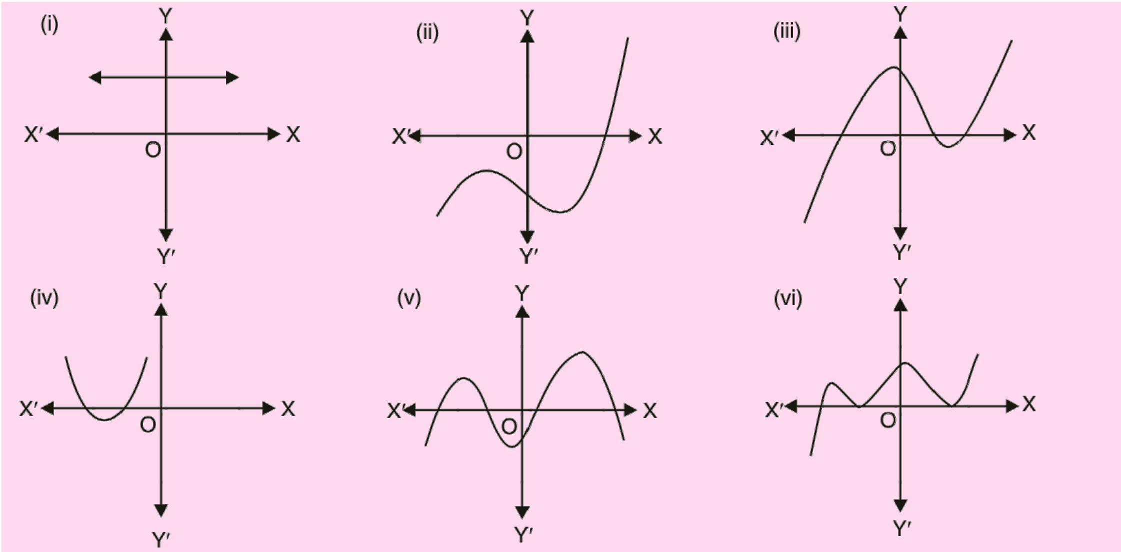
POLYNOMIALS

EXERCISE 2.1

QUESTION 1. ***The graphs of y = p(x) are given below, for some polynomials p(x). Find the number of zeroes of p(x), in each case.***



SOLUTION. ***(i)*** There are **no** zeroes as the graph does not intersect the x-axis.

(ii) The number of zeroes is **one** as the graph intersects the x-axis at one point only.

(iii) The number of zeroes is **three** as the graph intersects the x-axis at three points.

(iv) The number of zeroes is **two** as the graph intersects the x-axis at two points.

(v) The number of zeroes is **four** as the graph intersects the x-axis at four points.

(vi) The number of zeroes is **three** as the graph intersects the x-axis at three points. **Ans.**

EXERCISE 2.2

QUESTION 1. ***Find the zeroes of the quadratic polynomials and verify a relationship between zeroes and its coefficients.***

(i) $x^2 - 2x - 8$ ***(ii)*** $4s^2 - 4s + 1$ ***(iii)*** $6x^2 - 3 - 7x$

(iv) $4u^2 + 8u$ ***(v)*** $t^2 - 15$ ***(vi)*** $3x^2 - x - 4$

SOLUTION. ***(i)*** $x^2 - 2x - 8 = x^2 - 4x + 2x - 8 = x(x-4) + 2(x-4) = (x-4)(x+2)$

So, the value of $x^2 - 2x - 8$ is zero when $x - 4 = 0$ or $x + 2 = 0$, *i.e.*, when $x = 4$ or $x = -2$.

So, the zeroes of $x^2 - 2x - 8$ are **4, –2.**

$$\text{Sum of the zeroes} = 4 - 2 = 2 = \frac{-(-2)}{1} = \frac{-\text{ coefficient of } x}{\text{coefficient of } x^2} = \mathbf{2}$$

$$\text{Product of the zeroes} = 4(-2) = -8 = \frac{-8}{1} = \frac{\text{constant term}}{\text{coefficient of } x^2} = \mathbf{-8}$$ **Verified.**

(ii)

$$\begin{aligned} 4s^2 - 4s + 1 &= 4s^2 - 2s - 2s + 1 \\ &= 2s(2s-1) - 1(2s-1) \\ &= (2s-1)(2s-1) = (2s-1)^2 \end{aligned}$$

So, the value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, or $s = \frac{1}{2}$

Zeroes of the polynomial are $\mathbf{\frac{1}{2}, \frac{1}{2}}$

Sum of the zeroes $=\frac{1}{2}+\frac{1}{2}=1=-\left(\frac{-4}{4}\right)=\frac{-\text{coefficient of } s}{\text{coefficient of } s^2}=\mathbf{1}$

Product of the zeroes $=\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{\text{constant term}}{\text{coefficient of } s^2}=\mathbf{\frac{1}{4}}$ **Verified.**

(iii) We have : $6x^2-3-7x=6x^2-7x-3=6x^2-9x+2x-3$
$=3x(2x-3)+1(2x-3)=(3x+1)(2x-3)$
The value of $6x^2-3-7x$ is 0, when the value of $(3x+1)(2x-3)$ is 0, *i.e.*,
when $3x+1=0$ or $2x-3=0$, *i.e.*, when $x=-\frac{1}{3}$ or $x=\frac{3}{2}$.
$\therefore$ The zeroes of $6x^2-3-7x$ are $\mathbf{-\frac{1}{3}}$ and $\mathbf{\frac{3}{2}}$.

Therefore, sum of the zeroes $=-\frac{1}{3}+\frac{3}{2}=\frac{7}{6}=\frac{-(-7)}{6}=\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}=\mathbf{\frac{7}{6}}$

and product of zeroes $=\left(-\frac{1}{3}\right)\left(\frac{3}{2}\right)=\frac{-3}{6}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}=\mathbf{\frac{-3}{6}}$ **Verified.**

(iv) We have : $4u^2+8u=4u(u+2)$
The value of $4u^2+8u$ is 0, when the value of $4u(u+2)=0$, *i.e.*, when $u=0$ or $u+2=0$, *i.e.*, when $u=0$ or $u=-2$.
$\therefore$ The zeroes of $4u^2+8u$ are **0** and **– 2.**

Therefore, sum of the zeroes $=0+(-2)=-2=\frac{-8}{4}=\frac{\text{Coefficient of } u}{\text{Coefficient of } u^2}=\mathbf{-2.}$

and product of zeroes $=(0)(-2)=0=\frac{0}{4}=\frac{\text{Constant term}}{\text{Coefficient of } u^2}=\mathbf{0}$ **Verified.**

(v) We have: $t^2-15=(t-\sqrt{15})(t+\sqrt{15})$
The value of t^2-15 is 0, when the value of $(t-\sqrt{15})(t+\sqrt{15})$ is 0, *i.e.*, when $t-\sqrt{15}=0$ or $t+\sqrt{15}=0$,
i.e., when $t-\sqrt{15}$ or $t+\sqrt{15}$.
$\therefore$ The zeroes of t^2-15 are $\mathbf{\sqrt{15}}$ and $\mathbf{-\sqrt{15}}$.

Therefore, sum of the zeroes $=\sqrt{15}+(-\sqrt{15})=0=\frac{-0}{1}=\frac{-\text{Coefficient of } t}{\text{Coefficient of } t^2}=\mathbf{0}$

and product of the zeroes $=(\sqrt{15})(-\sqrt{15})=-15=\frac{-15}{1}=\frac{\text{Constant term}}{\text{Coefficient of } t^2}=\mathbf{-15}$ **Verified.**

(vi) We have : $3x^2-x-4=3x^2+3x-4x-4=3x(x+1)-4(x+1)=(x+1)(3x-4)$
The value of $3x^2-x-4$ is 0, when the value of $(x+1)(3x-4)$ is 0, *i.e.*, when $x+1=0$ or $3x-4=0$,
i.e., when $x=-1$ or $x=\frac{4}{3}$.
$\therefore$ The zeroes of $3x^2-x-4$ are **– 1** and $\mathbf{\frac{4}{3}}$.

Therefore, sum of the zeroes $=-1+\frac{4}{3}=\frac{-3+4}{3}=\frac{1}{3}=\frac{-(-1)}{3}=-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}=\mathbf{\frac{1}{3}}$

and product of the zeroes $=(-1)\left(\frac{4}{3}\right)=\frac{\text{Constant term}}{\text{Coefficient of } x^2}=\mathbf{\frac{-4}{3}}$ **Verified.**

QUESTION 2. ***Find a quadratic polynomial whose sum of zeroes and product of zeroes are respectively.***

(i) $\frac{1}{4},-1$ *(ii)* $\sqrt{2},\ \frac{1}{3}$ *(iii)* $0,\sqrt{5}$ *(iv)* 1,1 *(v)* $-\frac{1}{4},\frac{1}{4}$ *(vi)* 4, 1

SOLUTION. Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β. ...(1)

(*i*) Here, $\alpha+\beta=\frac{1}{4}$ and $\alpha\cdot\beta=-1$

Thus the polynomial formed $= x^2 -$ (Sum of zeroes) x + Product of zeroes $= x^2-\left(\frac{1}{4}\right)x-1 = x^2-\frac{x}{4}-1$

The other polynomials are $k\left(x^2-\frac{x}{4}-1\right)$

If $k = 4$, then the polynomial is $\mathbf{4x^2 - x - 4}$. **Ans.**

(*ii*) Here, $\alpha+\beta=\sqrt{2}$ $\alpha.\beta=\frac{1}{3}$

Thus the polynomial formed

$= x^2 -$ (Sum of zeroes) x + Product of zeroes

$= x^2-(\sqrt{2})x+\frac{1}{3}$ or $x^2-\sqrt{2}\,x+\frac{1}{3}$

Other polynomials are $k\left(x^2-\sqrt{2}\,x+\frac{1}{3}\right)$

If $k = 3$, then the polynomial is $\mathbf{3x^2 - 3\sqrt{2}x+1}$ **Ans.**

(*iii*) Here, $\alpha+\beta=0$ and $\alpha\cdot\beta=\sqrt{5}$

Thus the polynomial formed

$= x^2 -$ (Sum of zeroes)x + Product of zeroes $= x^2 - (0)x + \sqrt{5} = \mathbf{x^2+\sqrt{5}}$. **Ans.**

(*iv*) Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β. Then,

$$\alpha+\beta = 1 = \frac{-(-1)}{1} = \frac{-b}{a}$$

$$\alpha\beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If $a = 1$, then $b = -1$ and $c = 1$.

$\therefore$ One quadratic polynomial which satisfy the given conditions is $\mathbf{x^2 - x + 1}$. **Ans.**

(*v*) Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β. Then,

$$\alpha+\beta = -\frac{1}{4} = \frac{-1}{4} = \frac{-b}{a}$$

and $$\alpha\beta = \frac{1}{4} = \frac{c}{a}$$

If $a = 4$, then $b = 1$ and $c = 1$.

$\therefore$ One quadratic polynomial which satisfy the given conditions is $\mathbf{4x^2 + x + 1}$.

(*vi*) Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β. Then,

$$\alpha+\beta = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$$

and $$\alpha\beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If $a = 1$, then $b = -4$ and $c = 1$.

$\therefore$ One quadratic polynomial which satisfy the given conditions is $\mathbf{x^2 - 4x + 1}$. **Ans.**

EXERCISE 2.3

QUESTION 1. *Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each given of the following :*

(i) $p(x) = x^3 - 3x^2 + 5x - 3,\ g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5,\ g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6,\ g(x) = 2 - x^2$

SOLUTION. (*i*) Here, dividend and divisor are both in standard forms. So, we have :

$$
\begin{array}{r|lllll}
\multicolumn{1}{r}{} & \multicolumn{5}{l}{x - 3} \\
\cline{2-6}
x^2 - 2 & x^3 & -\ 3x^2 & +\ 5x & -\ 3 & \\
 & x^3\ - & & -\ 2x & & \\
 & - & & + & & \\
\cline{2-6}
 & & -\ 3x^2 & +\ 7x & -\ 3 & \\
 & & -\ 3x^2 & & +\ 6 & \\
 & & + & & - & \\
\cline{3-6}
 & & & 7x & -\ 9 & \\
\cline{3-6}
\end{array}
$$

$\therefore$ The **quotient is $x - 3$** and the **remainder is $7x - 9$.** **Ans.**

(***ii***) Here, the dividend is already in the standard form and the divisor is not in the standard form. It can be written as $x^2 - x + 1$.

We have :

$$
\begin{array}{r|lllll}
\multicolumn{1}{r}{} & \multicolumn{5}{l}{x^2 + x - 3} \\
\cline{2-6}
x^2 - x + 1 & x^4 & & -\ 3x^2 & +\ 4x & +\ 5 \\
 & x^4 & -\ x^3 & +\ x^2 & & \\
 & - & + & - & & \\
\cline{2-6}
 & & x^3 & -\ 4x^2 & +\ 4x & \\
 & & x^3 & -\ x^2 & +\ x & \\
 & & - & + & - & \\
\cline{3-6}
 & & & -\ 3x^2 & +\ 3x & +\ 5 \\
 & & & -\ 3x^2 & +\ 3x & -\ 3 \\
 & & & + & - & + \\
\cline{4-6}
 & & & & & 8 \\
\cline{4-6}
\end{array}
$$

$\therefore$ The **quotient is $x^2 + x - 3$** and the **remainder is 8.** **Ans.**

(***iii***) We have divisor $[-x^2 + 2]$ and divident : $x^4 - 5x - 6$

$$
\begin{array}{r|llll}
\multicolumn{1}{r}{} & \multicolumn{4}{l}{-x^2 - 2} \\
\cline{2-5}
-x^2 + 2 & x^4 & & -\ 5x & +\ 6 \\
 & x^4 & -\ 2x^2 & & \\
 & - & + & & \\
\cline{2-5}
 & & 2x^2 & -\ 5x & +\ 6 \\
 & & 2x^2 & & -\ 4 \\
 & & - & & + \\
\cline{3-5}
 & & & -\ 5x & +\ 10 \\
\cline{3-5}
\end{array}
$$

$\therefore$ The **quotient is $-x^2 - 2$** and the **remainder is $-5x + 10$.** **Ans.**

QUESTION 2. *Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial :*

(i) $t^2 - 3;\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1;\ 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1;\ x^5 - 4x^3 + x^2 + 3x + 1$

SOLUTION. *(i)* Let us divide $2t^4 + 3t^3 - 2t^2 - 9t - 12$ by $t^2 - 3$.

We have :

$$
\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\
t^2 - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & 2t^4 \qquad\quad - 6t^2 \\
 & - \qquad\qquad + \\
\hline
 & 3t^3 + 4t^2 - 9t \\
 & 3t^3 \qquad\ - 9t \\
 & - \qquad\qquad + \\
\hline
 & 4t^2 - 12 \\
 & 4t^2 - 12 \\
 & - \qquad + \\
\hline
 & 0 \\
\end{array}
$$

Since the remainder is 0, therefore, $\boldsymbol{t^2 - 3}$ **is a factor** of $2t^4 + 3t^3 - 2t^2 - 9t - 12$. **Ans.**

(ii) Let us divide $3x^4 + 5x^3 - 7x^2 + 2x + 2$ by $x^2 + 3x + 1$. We get,

$$
\begin{array}{r|l}
 & 3x^2 - 4x + 2 \\
x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\
 & 3x^4 + 9x^3 + 3x^2 \\
 & - \quad\ - \quad\ - \\
\hline
 & -4x^3 - 10x^2 + 2x \\
 & -4x^3 - 12x^2 - 4x \\
 & + \qquad + \qquad + \\
\hline
 & 2x^2 + 6x + 2 \\
 & 2x^2 + 6x + 2 \\
 & - \quad\ - \quad\ - \\
\hline
 & 0 \\
\end{array}
$$

Since the remainder is 0, therefore, $\boldsymbol{x^2 + 3x + 1}$ **is a factor** of $3x^4 + 5x^3 - 7x^2 + 2x + 2$ **Ans.**

(iii) Let us divide $x^5 - 4x^3 + x^2 + 3x + 1$ by $x^3 - 3x + 1$. We get,

$$
\begin{array}{r|l}
 & x^2 - 1 \\
x^2 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\
 & x^5 - 3x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -x^3 + 3x + 1 \\
 & -x^3 + 3x - 1 \\
 & + \qquad - \quad + \\
\hline
 & 2 \\
\end{array}
$$

Here, remainder is $2(\neq 0)$. Therefore, $\boldsymbol{x^3 - 3x + 1}$ **is not a factor** of $x^5 - 4x^3 + x^2 + 3x + 1$. **Ans.**

QUESTION 3. *Obtain all the zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.*

SOLUTION. Since two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$, $x = \sqrt{\frac{5}{3}}$, $x = -\sqrt{\frac{5}{3}}$

$\Rightarrow \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3}$ or $3x^2 - 5$ is a factor of the given polynomial. Now, we apply the division algorithm to the given polynomial and $3x^2 - 5$.

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
3x^2 - 5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & 3x^4 - 5x^2 \\
 & - \quad + \\
\hline
 & 6x^3 + 3x^2 - 10x - 5 \\
 & 6x^3 - 10x \\
 & - \quad + \\
\hline
 & 3x^2 - 5 \\
 & 3x^2 - 5 \\
 & - \quad + \\
\hline
 & 0
\end{array}
$$

First term of quotient is $\frac{3x^4}{3x^2} = x^2$

Second term of quotient is $\frac{6x^3}{3x^2} = 2x$

Third term of quotient is $\frac{3x^2}{3x^2} = 1$

So, $3x^4 + 6x^3 - 2x^2 - 10x - 5 = (3x^2 - 5)(x^2 + 2x + 1) + 0 = (3x^2 - 5)(x + 1)^2$

Quotient $= x^2 + 2x + 1 = (x + 1)^2$; Zeroes of $(x + 1)^2$ are $-1, -1$.

Hence, all its zeroes are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1, -1$ **Ans.**

QUESTION 4. ***On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find g(x).***

$$p(x) = x^3 - 3x^2 + x + 2$$

$$q(x) = x - 2 \text{ and } r(x) = -2x + 4$$

SOLUTION. By Division Algorithm, we know that

$$p(x) = q(x) \times g(x) + r(x)$$

Therefore, $x^3 - 3x^2 + x + 2 = (x - 2) \times g(x) + (-2x + 4)$

$\Rightarrow x^3 - 3x^2 + x + 2 + 2x - 4 = (x - 2) \times g(x)$

$\Rightarrow \quad g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$

On dividing $x^3 - 3x^2 + 3x - 2$ by $x - 2$, we get $g(x)$

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x - 2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & - \quad + \\
\hline
 & -x^2 + 3x - 2 \\
 & -x^2 + 2x \\
 & + \quad - \\
\hline
 & x - 2 \\
 & x - 2 \\
 & - \quad + \\
\hline
 & 0
\end{array}
$$

First term of $q(x) = \frac{x^3}{x} = x^2$

Second term of $q(x) = \frac{-x^2}{x} = -x$

Third term of $q(x) = \frac{x}{x} = 1$

Hence, $\mathbf{g(x) = x^2 - x + 1}$. **Ans.**

QUESTION 5. *Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and*

(i) deg p(x) = deg q(x) *(ii) deg q(x) = deg r(x)* *(iii) deg q(x) = 0*

SOLUTION. (*i*) Let $q(x) = 3x^2 + 2x + 6$, degree of $q(x) = 2$

$p(x) = 12x^2 + 8x + 24$, degree of $p(x) = 2$

Here, **deg p(x) = deg q(x)** **Ans.**

(*ii*) $p(x) = x^5 + 2x^4 + 3x^3 + 5x^2 + 2$

$q(x) = x^2 + x + 1$, degree of $q(x) = 2$

$g(x) = x^3 + x^2 + x + 1$

$r(x) = 2x^2 - 2x + 1$, degree of $r(x) = 2$

Here, **deg q(x) = deg r(x)** **Ans.**

(*iii*) Let $p(x) = 2x^4 + 8x^3 + 6x^2 + 4x + 12$

$q(x) = 2$, degree of $q(x) = 0$

$g(x) = x^4 + 4x^3 + 3x^2 + 2x + 1$

$r(x) = 10$

Here, **deg q(x) = 0.** **Ans.**

EXERCISE 2.4 (OPTIONAL)*

QUESTION 1. *Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case :*

(i) $2x^3 + x^2 - 5x + 2;\ \frac{1}{2}, 1, -2$ *(ii)* $x^3 - 4x^2 + 5x - 2;\ 2, 1, 1$

SOLUTION. (*i*) Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we get

$a = 2, b = 1, c = -5$ and $d = 2$.

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2 = \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2 = \frac{1+1-10+8}{4} = \frac{0}{4} = 0$$

$$p(1) = 2(1)^3 + (1)^2 - 5(1) + 2 = 2 + 1 - 5 + 2 = 0$$

$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2 = 2(-8) + 4 + 10 + 2 = -16 + 16 = 0$$

$\therefore$ $\mathbf{\frac{1}{2}}$, **1 and – 2** are the zeroes of $2x^3 + x^2 - 5x + 2$.

So, $\alpha = \frac{1}{2}$, $\beta = 1$ and $\gamma = -2$.

Therefore, $$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = \frac{1+2-4}{2} = -\frac{1}{2} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \left(\frac{1}{2}\right)(1) + (1)(-2) + (-2)\left(\frac{1}{2}\right) = \frac{1}{2} - 2 - 1 = \frac{1-4-2}{2} = \frac{-5}{2} = \frac{c}{a}$$

and $\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = -1 = \frac{-2}{2} = \frac{-d}{a}$ **Verified.**

(*ii*) Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we get

$a = 1, b = -4, c = 5$ and $d = -2$.

$$p(2) = (2)^3 - 4(2)^3 + 5(2) - 2 = 8 - 16 + 10 - 2 = 0$$

$$p(1) = (1)^3 - 4(1)^2 + 5(1) - 2 = 1 - 4 + 5 - 2 = 0$$

$\therefore$ **2, 1 and 1 are the zeroes of** $x^3 - 4x^2 + 5x - 2$.

So, $\alpha = 2$, $\beta = 1$ and $\gamma = 1$.

Therefore, $$\alpha + \beta + \gamma = 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$$

$$\alpha\alpha\gamma + \gamma\beta + \beta = (2)(1) + (1)(1) + (1)(2) = 2 + 1 + 2 = 5 = \frac{\mathbf{5}}{\mathbf{1}} = \frac{\boldsymbol{c}}{\boldsymbol{a}}$$

and $\alpha\beta\gamma = (2)(1)(1) = 2 = \dfrac{\mathbf{-(-2)}}{\mathbf{1}} = \dfrac{\boldsymbol{-d}}{\boldsymbol{a}}$ **Verified.**

QUESTION 2. ***Find a cubic polynomial with the sum, sum of the products of its zeroes taken two at a time, and the product of its zeroes as 2, – 7, – 14 respectively.***

SOLUTION. Let the cubic polynomial be $ax^3 + bx^2 + cx + d$, and its zeroes be α, β and γ.

Then, $\alpha + \beta + \gamma = 2 = \dfrac{-(-2)}{1} = \dfrac{-b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = -7 = \dfrac{-7}{1} = \dfrac{c}{a}$

and $\alpha\beta\gamma = -14 = \dfrac{-14}{1} = \dfrac{-d}{a}$

If $a = 1$, then $b = -2$, $c = -7$ and $d = 14$.

So, one cubic polynomial which satisfy the given conditions will be $\boldsymbol{x^3 - 2x^2 - 7x + 14}$. **Ans.**

QUESTION 3. ***If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b$, a and $a + b$, find a and b.***

SOLUTION. Since $(a - b)$, a and $(a + b)$ are the zeroes of the polynomial $x^3 - 3x^2 + x + 1$, therefore

$(a - b) + a + (a + b) = \dfrac{-(-3)}{1} = 3$

So, $3a = 3 \Rightarrow a = 1$

$(a - b)a + a(a + b) + (a + b)(a - b) = \dfrac{1}{1} = 1$

$\Rightarrow a^2 - ab + a^2 + ab + a^2 - b^2 = 1 \Rightarrow 3a^2 - b^2 = 1$

So, $3(1)^2 - b^2 = 1 \Rightarrow 3 - b^2 = 1$

$\Rightarrow b^2 = 2$ or $b = \pm\sqrt{2}$

Hence, $\boldsymbol{a = 1}$ and $b = \pm\sqrt{\mathbf{2}}$. **Ans.**

QUESTION 4. ***If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.***

SOLUTION. We have : $2 \pm \sqrt{3}$ are two zeroes of the polynomial

$p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

Let $x = 2 \pm \sqrt{3}$. So, $x - 2 = \pm\sqrt{3}$

Squaring, we get

$x^2 - 4x + 4 = 3$, *i.e.*, $x^2 - 4x + 1 = 0$

Let us divide $p(x)$ by $x^2 - 4x + 1$ to obtain other zeroes.

$$\begin{array}{r|rrrrr}
 & & x^2 & -2x & -35 & \\
\hline
x^2 - 4x + 1 & x^4 & -6x^3 & -26x^2 & +138x & -35 \\
 & x^4 & -4x^3 & +x^2 & & \\
 & - & + & - & & \\
\hline
 & & -2x^3 & -27x^2 & +138x & \\
 & & -2x^3 & +8x^2 & -2x & \\
 & & + & - & + & \\
\hline
 & & & -35x^2 & +140x & -35 \\
 & & & -35x^2 & +140x & -35 \\
 & & & + & - & + \\
\hline
 & & & & 0 & \\
\end{array}$$

$\therefore \quad p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

$= (x^2 - 4x + 1)(x^2 - 2x - 35)$

$= (x^2 - 4x + 1)(x^2 - 7x + 5x - 35)$

$= (x^2 - 4x + 1)[x(x - 7) + 5(x - 7)]$

$= (x^2 - 4x + 1)(x + 5)(x - 7)$

So, $(x + 5)$ and $(x - 7)$ are other factors of $p(x)$.

$\therefore$ ***– 5 and 7 are other zeroes of the given polynomial.*** **Ans.**

QUESTION 5. ***If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to $x + a$, find k and a.***

SOLUTION. Let us divide $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k$.

$$
\begin{array}{r|l}
 & x^2 - 4x + (8 - k) \\
\hline
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 25x + 10 \\
 & x^4 - 2x^3 + kx^2 \\
 & - \quad + \quad - \\
\hline
 & -4x^3 + (16 - k)x^2 - 25x \\
 & -4x^3 + 8x^2 - 4kx \\
 & + \quad - \quad + \\
\hline
 & (8 - k)x^2 + (4k - 25)x + 10 \\
 & (8 - k)x^2 - 2(8 - k)x + (8 - k)k \\
 & - \quad + \quad - \\
\hline
 & (2k - 9)x - (8 - k)k + 10
\end{array}
$$

$\therefore$ Remainder $= (2k - 9)\,x - (8 - k)\,k + 10$

But the remainder is given as $x + a$.

On comparing their coefficients, we have :

$2k - 9 = 1 \Rightarrow 2k = 10 \Rightarrow k = 5$

and $-(8 - k)k + 10 = a$

So, $a = -(8 - 5)5 + 10$

$= -3 \times 5 + 10 = -15 + 10 = -5$

Hence, $\boldsymbol{k = 5}$ and $\boldsymbol{a = -5}$. **Ans.**